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Testing the Adler and Gross - Llewellyn Smith Sum Rules in High Energy Neutrino Reactions*

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A practical method of testing the Adler and Gross - Llewellyn Smith sum rules with high energy neutrino beams is outlined. The test is found to rely only on a knowledge of the relative neutrino and anti-neutrino fluxes and does not require a point by point knowledge of the F_2 and F_3 structure functions. This test does rely on the assumption that Bjorken scaling is satisfied.

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Testing the Adler sum rule is generally recognized to be crucial to the application of presently accepted concepts of current algebra and constituent models at high energy.[1] Similarly the Gross-Llewellyn Smith (GLS) sum rule test specifically for constituents with Gell Mann-Zweig quark quantum numbers.[2] It is generally accepted that experimentally testing these sum rules will be extremely difficult and that the sum rules may converge slowly.[3] However, a closer look indicates that, provided Bjorken scaling holds, there are simple moments of experimentally accessible quantities that are directly related to the sum rules. Furthermore, in this case, it appears that a knowledge of the flux of neutrinos and anti-neutrinos is not necessary, but only the weaker condition that the relative ratio of neutrinos and anti-neutrinos is known. Indeed, the simultaneous production of neutrinos and anti-neutrinos using a broad band unfocussed beam allows the required knowledge of the relative flux. Using this method we anticipate that the Gross-Llewellyn Smith sum rule can be crudely tested in the near future using neutrino carbon and anti-neutrino carbon interactions in large calorimeter neutrino detectors.[4] Testing the Adler sum rule will require the introduction of a hydrogen target in such experiments.[4]

Consider the ratios of the form[5,6]

$$\langle f(Q^2, E_\mu)\rangle = \frac{\int f(Q^2, E_\mu)(d^2\sigma/dQ^2dE_\mu)dQ^2dE_\mu}{\int (d^2\sigma/dQ^2dE_\mu)dQ^2dE_\mu} \qquad (1)$$

where $\langle f\rangle$ can be chosen to be $\langle Q^2/2mE_\nu\rangle$, $\langle E_\mu/E_\nu\rangle$, etc. and are moments of the neutrino cross section distribution $d\sigma/dQ^2dE_\mu$ that are related to the experimental quantities

$$\langle f \rangle = \frac{\Sigma f_i N_i}{\Sigma N_i} \tag{2}$$

where N_i is the number of events for which $f = f_i$. The quantities $\langle f \rangle$ will be independent of the neutrino flux over the neutrino energy interval ΔE in the scaling limit. Using the scale invariant form of the neutrino and anti-neutrino cross section,[7]

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dxdy} = \frac{G^2 mE}{\pi}\left\{F_2(x)(1 - y) + \frac{1}{2}y^2[2xF_1(x)] \mp y(1 - \frac{1}{2}y)][xF_3(x)]\right\} \tag{3}$$

where

$$x = \frac{Q^2}{2m\nu} = \frac{1}{\omega} \text{ and } y = \frac{\nu}{E}$$

and where F_1, F_2 and F_3 are the neutrino structure functions and with the minus sign taken for neutrino interactions. We specify two different targets d and p where d refers to an $I = 0$ target with equal numbers of proton and neutron targets and p refers to a hydrogen target. We then consider two classes of structure functions

$$F_i^{\nu p}(x);\ F_i^{\bar{\nu}p}(x)$$

and

$$F_i^{\nu d}(x);\ F_i^{\bar{\nu}d}(x) \tag{4}$$

and it follows that (for $d = n + p$);

$$F_i^{\nu d} = F_i^{\bar{\nu}d} = F_i^{\nu p} + F_i^{\nu n} = F_i^{\bar{\nu}p} + F_i^{\bar{\nu}n}$$

and for a C^{12} target

$$F_i^{\nu C} = 6F_i^{\nu d}.$$

In the scaling limit the Adler and Gross-Llewellyn Smith sum rules can be expressed as

$$\int\frac{d\omega}{\omega}(F_2^{\nu n} - F_2^{\nu p}) = 2 \quad \text{(Adler)} \tag{5}$$

$$= \int\frac{d\omega}{\omega}(F_2^{\bar{\nu}p} - F_2^{\nu p}) = 2 \quad \text{(Adler)}$$

$$\int F_3^{\nu d}dx = \int\frac{d\omega}{\omega^2}(F_3^{\nu p} + F_3^{\nu n}) = \int\frac{d\omega}{\omega^2}(F_3^{\nu d}) = -6\text{(GLS)} \tag{6}$$

We consider the GLS sum rule first and form the mean values

$$\langle\omega\rangle_{\bar{\nu}d,\nu d} = \int\frac{d^2\sigma^{\bar{\nu}d,\nu d}}{dxdy}\,\frac{dxdy}{x}/\sigma_{\bar{\nu}d,\nu d} \tag{7}$$

Subtracting the quantity $(\sigma_{\nu d}/\sigma_{\bar{\nu}d})\langle\omega\rangle_{\bar{\nu}d}$ from $\langle\omega\rangle_{\nu d}$ gives

$$\left(\frac{\sigma_{\bar{\nu}d}}{\sigma_{\nu d}}\right)\langle\omega\rangle_{\bar{\nu}d} - \langle\omega\rangle_{\nu d} = \frac{\int\left[\frac{1}{x}\left(\frac{d\sigma^{\bar{\nu}d}}{dxdy} - \frac{d\sigma^{\nu d}}{dxdy}\right)\right]dxdy}{\sigma_{\nu d}} \tag{8}$$

$$= \frac{2\left(\int y(1 - \frac{1}{2}y)dy\right)\left\{\int F_3^{\nu d}dx\right\}}{\left[\frac{1}{2} + \frac{1}{2}\langle L\rangle - \frac{1}{6}\langle R\rangle\right]\int F_2^{\nu d}dx}$$

$$= \frac{2}{3}\,\frac{1}{\left[\frac{1}{2} + \frac{1}{2}\langle L\rangle - \frac{1}{6}\langle R\rangle\right]}\int F_3^{\nu d}dx,$$

where $\langle L\rangle$ and $\langle R\rangle$ are defined by Bjorken and Paschos[8], and $\sigma_{\nu d}$; $\sigma_{\bar{\nu}d}$ are the total cross sections.

In the scaling limit the ratio of total cross sections $\sigma_{\bar{\nu}d}/\sigma_{\nu d}$ becomes constant. Present CERN data suggests that this value is

$$\sigma_{\bar{\nu}d}/\sigma_{\nu d} = \frac{1}{3}(1 + \varepsilon) \tag{9}$$

where ε is less than 0.15.[9] For simplicity we derive illustrative results based on $\varepsilon \to 0$, keeping in mind that if ε is found to be larger at high energies the equations can be appropriately modified. The resulting equation is

$$(\sigma_{\bar{\nu}d}/\sigma_{\nu d})\langle\omega\rangle_{\bar{\nu}d} - \langle\omega\rangle_{\nu d} = \frac{2}{3}\int_0^1 F_3^{\nu d}dx \tag{10}$$

For Gell Mann-Zweig quark constituents we find

$$\langle\omega\rangle_{\nu d} - \frac{\sigma_{\bar{\nu}d}}{\sigma_{\nu d}}\langle\omega\rangle_{\bar{\nu}d} = 4 \tag{11}$$

and for unit baryon number constituents

$$\langle\omega\rangle_{\nu d} - \frac{\sigma_{\bar{\nu}d}}{\sigma_{\nu d}}\langle\omega\rangle_{\bar{\nu}d} = \frac{4}{3} \tag{12}$$

implying a considerable difference between the first moment of ω for the two cases. Verification of relation (11) requires only a knowledge of the ratio $\sigma_{\bar{\nu}d}/\sigma_{\nu d}$ and the moments of the ω distribution for $\bar{\nu}d$ and νd scattering, and, of course, that the sum rule converge in the accessible neutrino energy range.[10]

Turning to the Adler sum rule and assuming that the Callan-Gross relation[11,19]

$$F_2 = 2xF_1 \tag{13}$$

holds, we derive

$$\frac{2}{3}\int_0^1[F_2^{\nu n} - F_2^{\nu p}]\frac{dx}{x} + \frac{1}{2}R = \frac{(\sigma_{\nu p})}{(\sigma_{\nu d})}[\langle\omega\rangle_{\bar{\nu}p}\frac{(\sigma_{\bar{\nu}p})}{(\sigma_{\nu p})} - \langle\omega\rangle_{\nu p}] \tag{14}$$

where

$$R = (\sigma_{\bar{\nu}d}/\sigma_{\nu d})\langle\omega\rangle_{\bar{\nu}d} - \langle\omega\rangle_{\nu d} \tag{15}$$

Experimental measurement of R as well as the ratios $\sigma_{\nu p}/\sigma_{\nu d}$, $\sigma_{\bar{\nu}p}/\sigma_{\nu p}$ and the two moments of ω, $\langle\omega\rangle_{\bar{\nu}p}$ and $\langle\omega\rangle_{\nu p}$ allow a test of the Adler sum rule. Assuming both the (GLS) and Adler sum rules relation (14) can be rewritten as

$$(\frac{\sigma_{\nu p}}{\sigma_{\nu d}})[\langle\omega\rangle_{\nu p} - \frac{\sigma_{\bar{\nu}p}}{\sigma_{\nu p}}\langle\omega\rangle_{\bar{\nu}p}] = \frac{2}{3} \tag{16}$$

A direct measurement of $\sigma_{\nu p}/\sigma_{\nu d}$ can be made independent of neutrino flux if a hydrogen target and a carbon target are simultaneously exposed to the same neutrino beam, whereas the ratio $\sigma_{\bar{\nu}p}/\sigma_{\nu p}$ requires a hydrogen target exposed to a mixed neutrino-anti-neutrino beam of known ratio.

There are also an infinite number of sum rules given by

$$\frac{2}{3}\int x^n(F_2^{\nu d})dx + \frac{1}{3}\int x^{n+1}(F_3^{\bar{\nu}p} - F_3^{\nu p})dx$$
$$= \left(\frac{\sigma_{\nu p}}{\sigma_{\nu d}}\right)[\langle x^n\rangle_{\bar{\nu}p}(\sigma_{\bar{\nu}p}/\sigma_{\nu p}) + \langle x^n\rangle_{\nu p}] \tag{17}$$

Using the Llewellyn Smith relation[13] (which assumes non-integral charged quarks) with the Callan-Gross relation[11]

$$12[F_1^{\gamma p} - F_1^{\gamma n}] = \frac{6}{x}[F_2^{\gamma p} - F_2^{\gamma n}] = [F_3^{\nu p} - F_3^{\bar{\nu}p}] \tag{18}$$

relation (17) can be rewritten as

$$\frac{2}{3}\int x^n(F_2^{\nu d})d + 2\int x^n[F_2^{\gamma n} - F_2^{\gamma p}]dx$$
$$= \left(\frac{\sigma_{\nu p}}{\sigma_{\nu d}}\right)[\langle x^n\rangle_{\bar{\nu}p}\left(\frac{\sigma_{\bar{\nu}p}}{\sigma_{\nu p}}\right) + \langle x^n\rangle_{\nu p}] \tag{19}$$

Relation (19) implies that a direct experimental relation should obtain between moments of x measured in charged lepton scattering and the moments measured in neutrino interactions. The first two moments ($n = 0$ and $n = 1$) can be evaluated giving, for $n = 0$

$$\frac{2}{3}\int F_2^{\nu d}dx + 2\int[F_2^{\gamma n} - F_2^{\gamma p}]dx = \left(\frac{\sigma_{\bar{\nu}p}}{\sigma_{\nu d}}\right) + \left(\frac{\sigma_{\nu p}}{\sigma_{\nu d}}\right) \tag{20}$$

and for $n = 1$

$$\frac{2}{3}\int xF_2^{\nu d}dx + 2\int x[F_2^{\gamma n} - F_2^{\gamma p}]dx$$
$$= \left(\frac{\sigma_{\nu p}}{\sigma_{\nu c}}\right)[\langle x\rangle_{\bar{\nu}p}\left(\frac{\sigma_{\bar{\nu}p}}{\sigma_{\nu p}}\right) + \langle x\rangle_{\nu p}] \tag{21}$$

Using the best estimates for the neutrino integrals[8,14]

$$\int F_2^{\nu d}dx = 2(0.49 \pm .07) \tag{22}$$
$$= .98 \pm .14$$

and

$$\int xF_2^{\nu d}dx \simeq 2(.12) = .24 \tag{23}$$

and the electroproduction integrals[15]

$$\int (F_2^{\gamma p} - F_2^{\gamma n})dx = 0.05 \tag{24}$$

and

$$\int x(F_2^{\gamma p} - F_2^{\gamma n})dx = 0.016 \tag{25}$$

we find

$$\frac{\sigma_{\bar{\nu}p}}{\sigma_{\nu d}} + \frac{\sigma_{\nu p}}{\sigma_{\nu d}} = \frac{2}{3}(.98 \pm .14) + 2\int (F_2^{\gamma n} - F_2^{\gamma p})dx \tag{26}$$
$$\approx .55 \pm .09$$

and

$$\frac{\sigma_{\nu p}}{\sigma_{\nu d}}[\langle x\rangle_{\bar{\nu}p}(\frac{\sigma_{\bar{\nu}p}}{\sigma_{\nu p}}) + \langle x\rangle_{\nu p}] \simeq .16 + 2\int x(F_2^{\gamma n} - F_2^{\gamma p})dx$$
$$\simeq .13 \tag{27}$$

Experimental verification of relations (26) and (27) would constitute a partial test of the Llewellyn Smith relation (18), implying a point by point relation between the structure functions measured in electromagnetic and weak lepton inclusive interactions.

Although the testing of the Adler and GLS sum rules will undoubtedly be extremely difficult, it is not necessary to separate the structure function point by point but instead only the first moments of the distribution and some relative cross sections measurements are required. In addition, there exists an infinite set of sum rules relating the moments of the x distributions measured in electromagnetic and weak scattering processes, provided the Llewellyn Smith relation holds. Testing these sum rules would constitute an important test of the nonintegral charge quark model. Even if relation (18) were found to be incorrect experimentally, it is still expected that a similar relation between the weak and electromagnetic structure functions might hold and that a relation analogous to (19) would be discovered. Clearly a parameterization of the experimental data in terms of moments would be important in this regard.

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